Holography, localization, black holes

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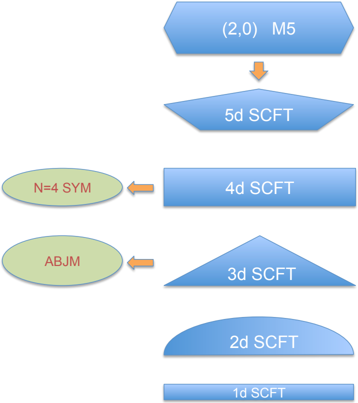
Introduction

Lot of recent activity in the study of supersymmetric and superconformal theories in various dimensions in closely related contexts

- A deeply interconnected web of supersymmetric theories arising from branes, most often strongly coupled, related by various types of dualities.
- Progresses in the evaluation of exact quantum observables. Related to localization and the study of supersymmetry in curved space.
- Many results on indices and counting problems that can be also related to BH physics.

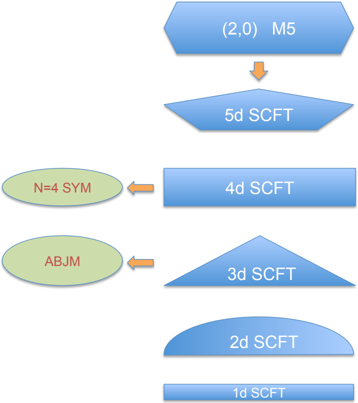
Introduction

A large superconformal zoo



Introduction

A large superconformal zoo with holographic duals



AdS_7

AdS_6

AdS_5

AdS_4

AdS_3

AdS_2

Introduction

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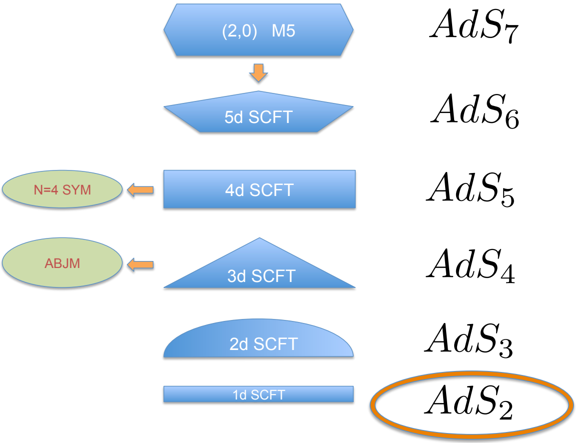


Introduction

Supersymmetric black holes have horizon $\text{AdS}_2 \times S^2$

- CFT_1 living at the horizon of black holes
- density of states $=$ BH entropy

Introduction

Conformal algebra in 1d involves $SL(2, R)$

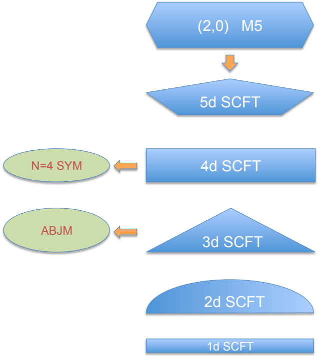
$$H\,, \qquad D, \qquad K$$

with various superconformal extensions.

- AdS_2/CFT_1 still poorly understood.

Introduction

Supersymmetric AdS black holes are also related by holography to states of CFT_3 and CFT_4



AdS_5 rotating BH

AdS_4 static BH

Introduction

Entropy of AdS BH should be related to the counting of supersymmetric states in CFT:

- We still don't know where to look for AdS_5: people tried with superconformal index, 1/4-BPS partition functions, ...
- We have now an answer for AdS_4: the index of the topologically twisted theory

Euclidean SCFT in finite volume

To regularize IR physics we can put the theory on a Euclidean compact manifold. Supersymmetry restricts the manifold

Euclidean SCFT in finite volume

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Topological Field Theories [Witten, 1988]

- $N = 2$ theories in 4d with $SU(2)_R$ R-symmetry can be consistently defined on any M_4 euclidean manifold
- The resulting theory is *topological*: independent of metric.

$$T_{\mu\nu} = \{Q, \cdots\}$$

Euclidean SCFT in finite volume

To regularize IR physics we can put the theory on a Euclidean compact manifold. Supersymmetry restricts the manifold

SCFT on Spheres [Pestun 2007]

- Just use conformal map from $\mathbb{R}^n$ to S^n

Euclidean SCFT in finite volume

To regularize IR physics we can put the theory on a Euclidean compact manifold. Supersymmetry restricts the manifold

In between a pletora of possibilities.

- A general method is to couple to supergravity and find solution of [Festuccia-Seiberg]

$$\delta\psi_\mu(x) = \nabla_\mu \epsilon + \cdots \equiv 0$$

Backgrounds with twisted (conformal) Killing spinors have been classified in various dimensions and with various amount of supersymmetry.

[klare,tomasiello,A.Z.;dumitruescu,festuccia,seiberg;Closser,Dumitrescu,Festuccia,Komargodski; ...]

Superconformal theories on curved spaces

For SCFT, the variation of the gravitino can be written as a (twisted) conformal Killing equation

$$(\nabla_a - iA_a^R)\epsilon_+ + \gamma_a \epsilon_- = 0 \qquad \Longrightarrow \qquad \nabla_a^A \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+$$

Superconformal theories on curved spaces

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- for theories with more supercharges and in different dimensions, A_a^R is promoted to a non-abelian gauge field, there can be other backgrounds tensor fields and extra conditions.

Superconformal theories on curved spaces

A possible solution is to have covariantly constant spinors

$$\nabla_a^A \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+ \qquad \text{solved by} \qquad \nabla_a^A \epsilon_+ = 0$$

Topological Field Theories can be realized this way by canceling the spin connection with a background R-symmetry

$$\nabla_a^A \epsilon_+ = \partial_a \epsilon_+ + \frac{1}{4}\omega_a^{\alpha\beta}\Gamma^{\alpha\beta}\epsilon_+ + A_a^R \epsilon_+ = \partial_a \epsilon_+$$

which can be solved by $\epsilon_+ =$ constant on any manifold

Superconformal theories on curved spaces

The well know examples of Topological Field Theories back in the eighties

$$\nabla_a^A \epsilon_+ = \partial_a \epsilon_+ + \frac{1}{4}\omega_a^{\alpha\beta}\Gamma^{\alpha\beta}\epsilon_+ + A_a^R \epsilon_+ = \partial_a \epsilon_+$$

- $N = 2$ theories on any M_4: $SU(2)_R$ background gauge field

  ϵ_+ transform in the $(\mathbf{2}, \mathbf{0})$ representation of the local Euclidean group $SO(4) = SU(2) \times SU(2)$

- A twist in 2d on any Riemann surface Σ_g: $U(1)_R$ background gauge field

Superconformal theories on curved spaces

A possible solution is to have Killing spinors

$$\nabla_a^A \epsilon_+ = \frac{1}{d}\gamma_a \not\nabla^A \epsilon_+ \qquad \text{solved by} \qquad \nabla_a \epsilon_+ = \gamma_a \epsilon_+$$

Realized for Theories on Spheres, with zero background field $A^R = 0$.

Superconformal theories on curved spaces

Or combinations: Killing spinors on S^d and R- and flavor holonomies on S^1

$$\nabla_a^A \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+ \qquad \text{solved by} \qquad \nabla_a \epsilon_+ = \gamma_a \epsilon_+ \, , A_t = i$$

Superconformal theories on curved spaces

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$$\nabla^A_a \epsilon_+ = \frac{1}{d}\gamma_a \nabla\!\!\!\!/^A \epsilon_+ \qquad \text{solved by} \qquad \nabla_a \epsilon_+ = \gamma_a \epsilon_+ \, , A_t = i$$

The path integral on $S^d \times S^1$ can be written as a trace over a Hilbert space

$$Z_{S^d \times S^1} = \mathrm{Tr}(-1)^F e^{-\beta\{Q,S\}+\sum \Delta_a J_a}$$

This is the superconformal index: it counts BPS states graded by dimension and charges

[Romelsberger; Kinney,Maldacena,Minwalla,Rahu]

computed for large class of 4d SCFT [Gadde,Rastelli,Razamat,Yan]

Superconformal theories on curved spaces

Or combinations: a topological twist on Σ_g and flavor holonomies on S^1 (or T^2)

$$\nabla^A_a \epsilon_+ = \frac{1}{d}\gamma_a \not{\nabla}^A \epsilon_+ \qquad \text{solved by} \qquad \nabla^A_a \epsilon_+ = 0\,,\, A^R_a = \omega_a$$

The integral $\int_{\Sigma_g} F^R$ gives a magnetic charge for R-symmetry

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The integral $\int_{\Sigma_g} F^R$ gives a magnetic charge for R-symmetry

The path integral on $\Sigma_g \times S^1$ can be written as a Witten index (elliptic genus)

$$Z_{\Sigma_g \times S^1} = \mathrm{Tr}_{\mathcal{H}} \left((-1)^F e^{iJ_F A^F} e^{-\beta H} \right)$$

$Q^2 = H - \sigma^F J_F$

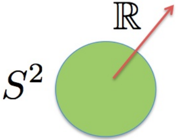
holomorphic in $A^F + i\sigma^F$

of the dimensional reduced theory on Σ_g and counts ground states graded by charges. This is the topologically twisted index.

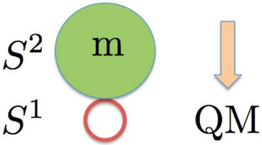
[Benini,AZ; Closset-Kim]

Superconformal theories on curved spaces

The two indices are quite different



- the superconformal index counts the BPS states on S^2/operators



- the topologically twisted index counts the ground states/Landau levels of the theory on S^2 with magnetic charges for R and flavor symmetries

Localization

Exact quantities in supersymmetric theories with a charge $Q^2 = 0$ can be obtained by a saddle point approximation

$$Z = \int e^{-S} = \int e^{-S+t\{Q,V\}} \underset{t\gg 1}{=} e^{-\bar{S}|_{class}} \times \frac{\det_{fermions}}{\det_{bosons}}$$

$$\partial_t Z = \int \{Q, V\} e^{-S+t\{Q,V\}} = 0$$

Very old idea that has become very concrete recently, with the computation of partition functions on spheres and other manifolds supporting supersymmetry.

Localization

Localization ideas apply to path integral of Euclidean supersymmetric theories

- Compact space provides IR cut-off, making path integral well defined
- Localization reduces it to a finite dimensional integral, a matrix model

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- Localization reduces it to a finite dimensional integral, a matrix model

$$\int \prod_{i=1}^{N_1} du_i \prod_{j=1}^{N_2} dv_j \frac{\prod_{i<j} \sinh^2 \frac{u_i-u_j}{2} \sinh^2 \frac{v_i-v_j}{2}}{\prod_{i<j} \cosh^2 \frac{u_i-v_j}{2}} e^{\frac{ik}{4\pi}\left(\sum u_i^2-\sum v_j^2\right)}$$

ABJM, 3d Chern-Simon theories, [Kapustin,Willet,Yakoov;Drukker,Marino,Putrov]

Localization

Carried out recently in many cases

- many papers on topological theories
- S^2, T^2
- S^3, $S^3/\mathbb{Z}_k$, $S^2 \times S^1$, Seifert manifolds
- S^4, $S^4/\mathbb{Z}_k$, $S^3 \times S^1$, ellipsoids
- S^5, $S^4 \times S^1$, Sasaki-Einstein manifolds

with addition of boundaries, codimension-2 operators, ...

Pestun 07; Kapustin,Willet,Yakoov; Kim; Jafferis; Hama,Hosomichi,Lee, too many to count them all $\cdots$

Localization

In all cases, it reduces to a finite-dimensional matrix model on gauge variables, possibly summed over different topological sectors

$$Z_M(y) = \sum_{\mathfrak{m}} \int_C dx\, Z_{\mathrm{int}}(x, y; \mathfrak{m})$$

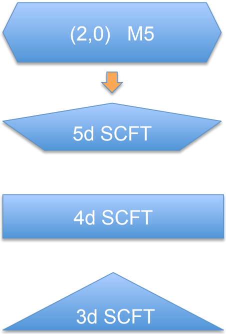
with different integrands and integration contours.

When backgrounds for flavor symmetries are introduced, $Z_M(y)$ becomes an interesting and complicated function of y which can be used to test dualities

- Sphere partition function, Kapustin-Willet-Yakoov; $\cdots$
- Superconformal index, Spironov-Vartanov; Gadde,Rastelli,Razamat,Yan; $\cdots$
- Topologically twisted index, Benini,AZ; Closset-KIm; $\cdots$

Free energy on S^d

Successfully testing holographic prediction at large N: large N matrix model techniques



$$AdS_7 \quad \rightarrow \quad \mathcal{F} \sim N^3$$

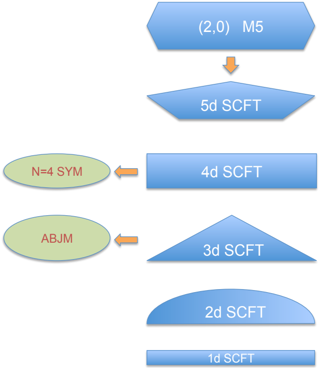
$$AdS_6 \quad \rightarrow \quad \mathcal{F} \sim N^{5/2}$$

$$AdS_5 \quad \rightarrow \quad \mathcal{F} \sim N^2$$

$$AdS_4 \quad \rightarrow \quad \mathcal{F} \sim N^{3/2}, \quad N^{5/3} \ldots$$

Free energy on S^d

The free energy on S^3 filled a gap

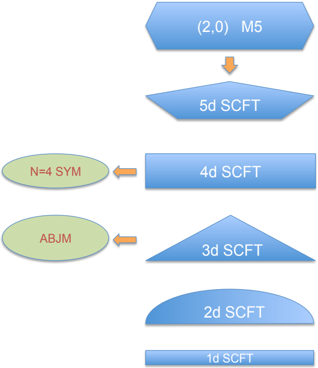


$a-$maximization

$c-$maximization

Free energy on S^d

The free energy on S^3 filled a gap [Jafferis,Klebanov,Pufu,Safdi;Casini,Huerta]



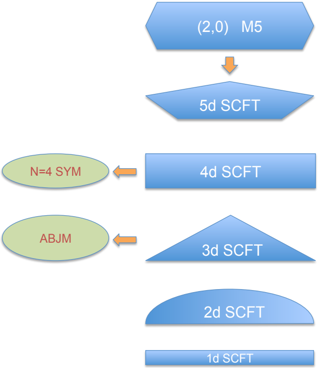
a − maximization

F_{S^3} − extremization

c − maximization

Free energy on S^d

The free energy on S^3 filled a gap



$a-$ maximization

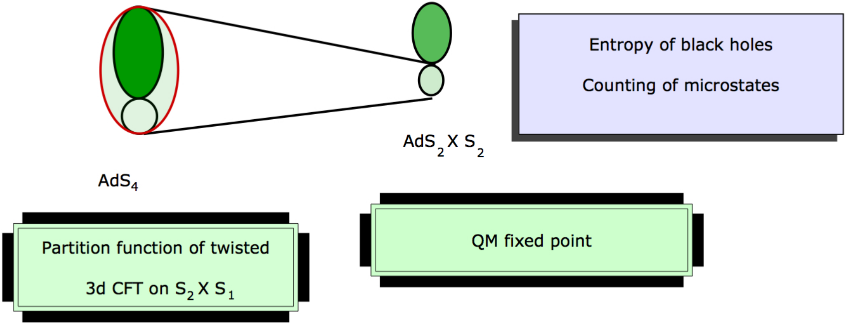
$F_{S^3}-$ extremization

$c-$ maximization

Witten index?

AdS_4 black holes

A nice arena where many of the previous ingredients meet is the counting of microstates of asymptotically AdS_4 BPS black holes [Benini,Hristov, AZ]



AdS$_4$ black holes

I'm talking about BPS asymptotically AdS$_4$ static black holes

$$\mathrm{d}s^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 \mathrm{d}t^2 - \frac{e^{-\mathcal{K}(X)} \mathrm{d}r^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 \mathrm{d}s^2_{\Sigma_g}$$

- vacua of $N = 2$ gauged supergravities arising from M theory truncations
- supported by magnetic charges on Σ_g: $\mathfrak{n} = \frac{1}{2\pi} \int_{\Sigma_g^2} F$
- preserving supersymmetry via an R-symmetry twist

$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu \epsilon \qquad \Longrightarrow \qquad \epsilon = \text{cost}$$

[Cacciatori,Klemm; Gnecchi,Dall'agata; Hristov,Vandoren;Halmagyi;Katmadas]

Holographic Perspective

General vacua of a bulk effective action

$$\mathcal{L} = -\frac{1}{2}\mathcal{R} + F_{\mu\nu}F^{\mu\nu} + V\ldots$$

with a metric

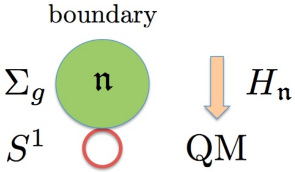
$$ds^2_{d+1} = \frac{dr^2}{r^2} + (r^2 ds^2_{M_d} + O(r)) \qquad A = A_{M_d} + O(1/r)$$

and a gauge fields profile, correspond to CFTs on a d-manifold M_d and a non trivial background field for the R- or global symmetry

$$L_{CFT} + J^\mu A_\mu$$

AdS$_4$ black holes

The boundary theory is topologically twisted, with a magnetic charge for the R-symmetry and for the global symmetries of the theory



$$Z^{\text{twisted}}_{\Sigma_g^2 \times S^1}(\mathfrak{n}, \Delta) = \text{Tr}_{\mathcal{H}} \left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{n}}} \right)$$

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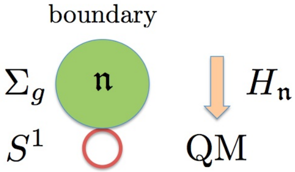

$$Z^{\text{twisted}}_{\Sigma_g^2 \times S^1}(\mathfrak{n}, \Delta) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{n}}}\right)$$

This is the Witten index of the QM obtained by reducing $\Sigma_g^2 \times S^1 \to S^1$.

- magnetic charges $\mathfrak{n}$ are not vanishing at the boundary and appear in the Hamiltonian
- electric charges can be introduced using chemical potentials Δ

AdS$_4$ black holes

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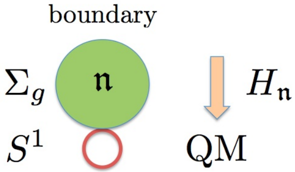


$$Z^{\text{twisted}}_{\Sigma_g^2\times S^1}(\mathfrak{n},\Delta) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{n}}}\right)$$

The BH entropy is related to a Legendre Transform of the index [Benini-Hristov-AZ]

$$S_{BH}(\mathfrak{q},\mathfrak{n}) \equiv \mathbb{R}\mathrm{e}\,\mathcal{I}(\Delta) = \mathbb{R}\mathrm{e}(\log Z(\mathfrak{n},\Delta) - i\Delta\mathfrak{q})\,, \qquad \frac{d\mathcal{I}}{d\Delta} = 0$$

[similar to Sen's formalism, OSV, etc]

Black holes in AdS$_4 \times S^7$

The ABJM twisted index is

$$Z = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\widetilde{\mathfrak{m}} \in \mathbb{Z}^N} \int \prod_{i=1}^N \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} x_i^{k\mathfrak{m}_i} \tilde{x}_i^{-k\widetilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^N \left(1 - \frac{x_i}{x_j}\right)\left(1 - \frac{\tilde{x}_i}{\tilde{x}_j}\right) \times$$

$$\times \prod_{i,j=1}^N \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_1}}{1 - \frac{x_i}{\tilde{x}_j} y_1}\right)^{\mathfrak{m}_i - \widetilde{\mathfrak{m}}_j - \mathfrak{n}_1 + 1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_2}}{1 - \frac{x_i}{\tilde{x}_j} y_2}\right)^{\mathfrak{m}_i - \widetilde{\mathfrak{m}}_j - \mathfrak{n}_2 + 1}$$

$$\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_3}}{1 - \frac{\tilde{x}_j}{x_i} y_3}\right)^{\widetilde{\mathfrak{m}}_j - \mathfrak{m}_i - \mathfrak{n}_3 + 1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_4}}{1 - \frac{\tilde{x}_j}{x_i} y_4}\right)^{\widetilde{\mathfrak{m}}_j - \mathfrak{m}_i - \mathfrak{n}_4 + 1}$$

$$\prod_i y_i = 1 \,, \qquad \sum \mathfrak{n}_i = 2(1-g)$$

where $\mathfrak{m}, \widetilde{\mathfrak{m}}$ are the gauge magnetic fluxes, $y_i = e^{i\Delta_i}$ are fugacities and $\mathfrak{n}_i$ the magnetic fluxes for the three independent $U(1)$ global symmetries

Black holes in AdS$_4 \times S^7$

Strategy:

- Re-sum geometric series in $\mathfrak{m}, \widetilde{\mathfrak{m}}$.

$$Z = \int \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} \frac{f(x_i, \tilde{x}_i)}{\prod_{j=1}^N (e^{iB_i} - 1) \prod_{j=1}^N (e^{i\widetilde{B}_j} - 1)}$$

- Step 1: find the zeros of denominator $e^{iB_i} = e^{i\tilde{B}_j} = 1$ at large N
- Step 2: evaluate the residues at large N

$$Z \sim \sum_I \frac{f(x_i^{(0)}, \tilde{x}_i^{(0)})}{\det \mathbb{B}}$$

[Benini-Hristov-AZ]

[extended to other model Hosseini-AZ; Hosseini-Mekareeya]

Black holes in AdS$_4 \times S^7$

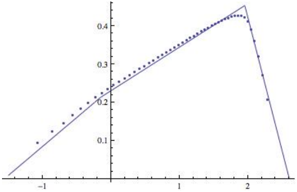
Step 1: solve the large N Limit of algebraic equations giving the positions of poles

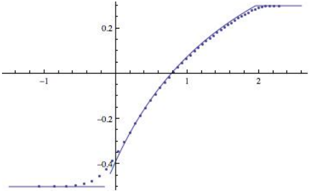
$$1 = x_i^k \prod_{j=1}^{N} \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})}$$

Bethe Ansatz Equations - derived by a potential $\mathcal{V}_{BA}(x_i, \tilde{x}_i)$

with an ansatz

$$\log x_i = i\sqrt{N} t_i + v_i \, , \qquad \log \tilde{x}_i = i\sqrt{N} t_i + \tilde{v}_i$$





Black holes in AdS$_4 \times S^7$

The index is obtained from $\mathcal{V}_{BA} \sim \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$:

$$\mathcal{I}(\Delta) = \frac{1}{3}N^{3/2}\sum_i\left(-\sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4}\,\frac{\mathfrak{n}_i}{\Delta_i} - i\Delta_i\mathfrak{q}_i\right) \qquad y_i = e^{i\Delta_i}$$

This function can be extremized with respect to the Δ_i and

$$\boxed{\mathbb{R}\mathrm{e}\,\mathcal{I}|_{crit} = \mathrm{BH\,Entropy}(\mathfrak{n}_i,\mathfrak{q}_i)}$$

$$\boxed{\Delta_i|_{crit} \sim X^i(r_h)}$$

[Benini-Hristov-AZ]

A. Attractor mechanism

The BPS equations at the horizon imply that the gauge supergravity quantity

$$\mathcal{R} = \left(F_\Lambda \mathfrak{n}^\Lambda - X^\Lambda \mathfrak{q}_\Lambda \right) , \qquad F_\Lambda = \frac{\partial \mathcal{F}}{\partial X^\Lambda}$$

with $(\mathfrak{q}, \mathfrak{n})$ electric and magnetic charges, is extremized with respect to the scalar fields at the horizon and its critical value gives the entropy

Under $X^\Lambda \to \Delta^\Lambda$

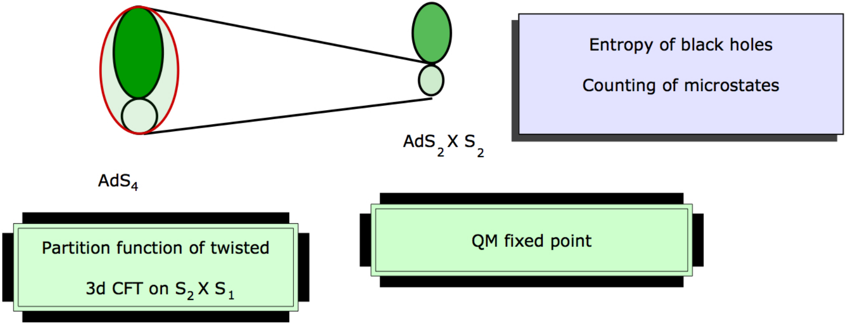
$$\mathcal{F} = 2i\sqrt{X^0 X^1 X^2 X^3} \equiv \mathcal{V}_{BA}(\Delta)$$

$$i\mathcal{R} = \sum -\frac{\mathfrak{n}_\Lambda}{X^\Lambda}\sqrt{X^0 X^1 X^2 X^3} - iX^\Lambda \mathfrak{q}_\Lambda \equiv \mathcal{I}(\Delta)$$

[Benini-Hristov-AZ; Hosseini-AZ]

B. R-symmetry extremization

Recall the cartoon



B. R-symmetry extremization

The extremization reflects exactly what's going on in the bulk. The graviphoton field strength depends on r

$$T_{\mu\nu} = e^{K/2} X^{\Lambda} F_{\Lambda,\mu\nu}$$

suggesting that the R-symmetry is different in the IR and indeed

$$\boxed{\Delta_i|_{crit} \sim X^i(r_h)}$$

B. R-symmetry extremization

Some QFT extremization is at work? symmetry enhancement at the horizon AdS_2

$$\mathrm{QM}_1 \to \mathrm{CFT}_1$$

The twisted index depends on Δ_i because we are computing the trace

$$\mathrm{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_i J_i} \equiv \mathrm{Tr}_{\mathcal{H}}(-1)^R$$

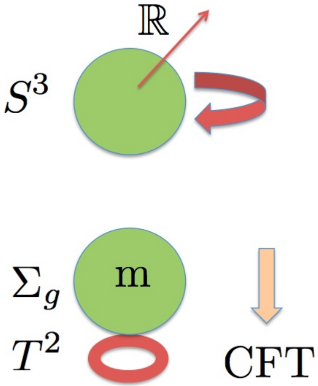
where $R = F + \Delta_i J_i$ is a possible R-symmetry of the system.

- R is the exact R-symmetry at the superconformal point
- Natural thing to extremize: in even dimensions central charges are extremized, in odd partition functions...

C. One dimension more

In AdS_5 there are two interesting objects

boundary



bulk

- AdS_5 rotating black hole; where the entropy comes from?

- AdS_5 black string; horizon $AdS_3 \times \Sigma_g$. 2d central charge of the CFT matched with gravity. c-extremization for R-symmetry

  [Benini-Bobev]

Conclusions

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Thank you for the attention !